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Case Study 3: Potato

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Change in Temperature in Baked Potato with and without Aluminum Foil

1. Introduction

In this case study, the temperature of a potato was taken as it cooled down right after being baked. Two potatoes were modeled and examined for this experiment. The first potato was tightly wrapped in aluminum foil while the second was unwrapped. Both were exposed to ambient room temperature. The main focus of this experiment was to see if the temperature of both potatoes could be accurately modeled over time. Both potatoes are subject to low biot and fall within the dunking problem class. Both potatoes main method of cooling is via natural convection and radiation, albeit the wrapped potato will radiate differently than the unwrapped due to different emissivities.

In the case of both potatoes, the shape of the potato was ambiguous and could fall between spherical or cylindrical. For the purpose of this study, the potatoes were both modeled and analyzed as spheres and cylinders. The cylindrical dimensions were found by measuring the diameter and length of the potato. The spherical dimensions were found by solving for the radius of the sphere that would make the surface area of the sphere and the surface area of the cylinder equal. Overall, while the potatoes of this experiment were shaped more cylindrically, this dual model allows easy implementation of spherical potatoes.

With regard to the modes of heat transfer, both potatoes going to placed into the lumped linear dunking problem class. They are both subject to natural convection, and the heat transfer coefficient for both cases of wrapped and unwrapped will remain unchanged. However, the heat transfer coefficient will change between spherical and cylindrical given different nusselt correlations. For radiation, the radiation is being emitted from the aluminum foil for the wrapped potato, and therefore will have a different emittance than the unwrapped potato. The radiation heat transfer coefficient will not change between spherical and cylindrical forms, but will change therefore between unwrapped and wrapped. Using the lumped linear model, predictions of the two potatoes temperatures were made at different time points.

The most applicable case of this case study is in the kitchen. Broadly, this case study will shed light on how drastically aluminum foil effects the cooling down or heating up of an item to room temperature. In this specific instance, it is important to know how long a baked potato will be warm, and therefore able to serve, after being taken out of the oven.

2. Experimental Setup

The first step in this setup was to preheat the oven to 420 degrees Fahrenheit. After the oven has preheated, both potatoes were cooked side by side for the same amount of time for 45 minutes. Immediately after being removed from the oven, one potato was wrapped in aluminum foil tightly and then both were placed on paper towels on a metal pan. Using a cooking thermometer, the temperature at approximately the center of the potato was measured over time. These readings were taken initially at 30 second intervals and then were expanded to eventually 5 minute intervals. The total time for taking measurements was 30 minutes. Shown in figure 2.1 and 2.2 are the temperatures of both potatoes over time.

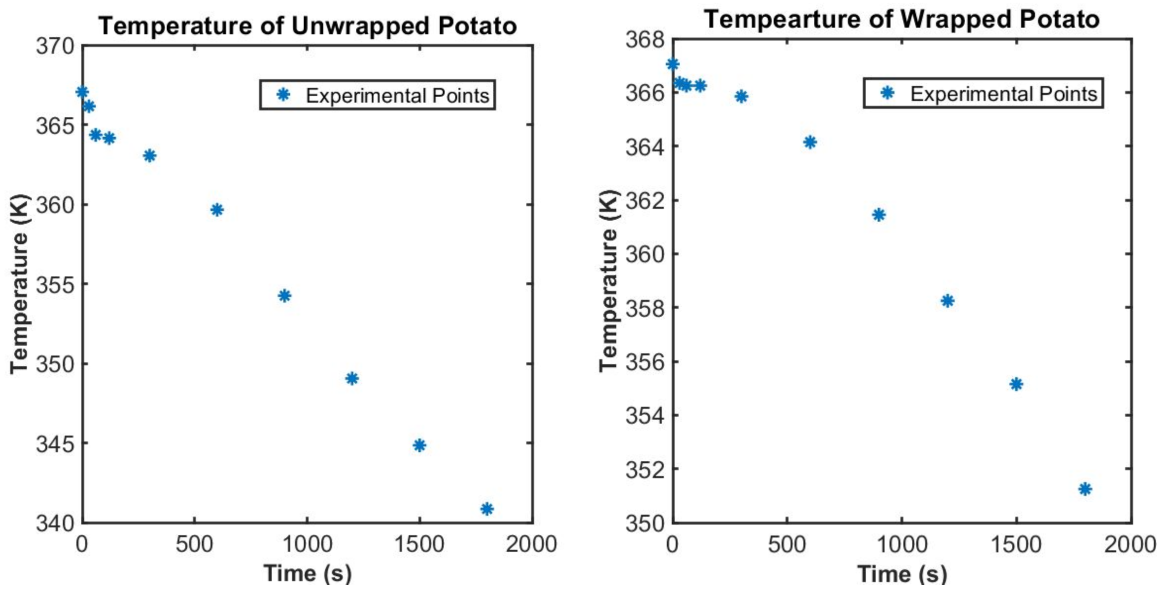

Figure 2.1 and 2.2

Both potatoes begin at 93.9 degrees C but cool down at different rates. After thirty minutes, the unwrapped potato temperature was 67.7 degrees C while the wrapped potato was measured to be 78.1 degrees C. The experiment confirms quantitatively that wrapping the potato in aluminum foil will keep it hot for longer. As can be seen, after thirty minutes, the difference in temperature between the wrapped and unwrapped potato is about 10 degrees C.

3. Analysis

As mentioned above, the analysis of both potatoes was in the linear lumped dunking problem class. To begin, the shape of the potatoes must be normalized to fit into either the spherical or cylindrical case. In this case study, analysis is done on both cases. For the cylindrical case, the diameter and length are taken with reference to the center of the potato. For the spherical case, the surface areas of the measured cylinder is equated to the surface area of a sphere for which the diameter is solved for. This is shown in equations (1).

$$\pi D_s^2 = \pi D_{cyl} L + \pi D_{cyl}^2/2 \quad (1)$$

Given that the diameter of the potato was measured to be $D_{cyl} = 0.606$ m and the length to be $L = 0.1458\ m$, the spherical diameter was found to be $D_s = 0.1033\ m$. Now that the models have been solidified in terms of physical properties, the heat transfer coefficients for natural convection and radiation must be found. For natural convection, the heat transfer coefficient will not change between the wrapped and unwrapped case. Therefore, the heat transfer coefficient will just be found for the spherical and cylindrical case. Beginning with the cylindrical case, the Nusselt and Rayleigh correlation can be used to find the coefficient as shown in equations (2), (3), and (4). For the spherical case, equations (2), (4), and (5) can be used to find the heat transfer coefficient.

$$Ra_D = g\beta D^3(T_i - T_{inf})/(\alpha * \nu)\ \textbf{(2)}$$

$$Nu_{D,\ cyl} = 0.36\ +\ .518Ra_D^{.25} * (1 + (0.559/Pr)^{9/16})^{-4/9}\ \textbf{(3)}$$

$$Nu_{D,\ sphere} = 2 + 0.589Ra_D^{.25} * (1 + (0.492/Pr)^{9/16})^{-4/9}\ \textbf{(4)}$$

$$h_{conv} =\ Nu_D\ * k_{air}/D\ \textbf{(5)}$$

Using these equations, the heat transfer coefficients are found to be $h_{conv,\ cylinder} =\ 6.097W/m^2K$ and $h_{conv,\ sphere} =\ 6.507W/m^2K$. Next, the heat transfer coefficient for radiation must be solved for. In this case, there is no difference between spherical and cylindrical, but there will be a difference between wrapped and unwrapped. Using equations (6), the heat transfer coefficient can be solved for in both cases. With regards to the temperature chosen for the linear model, the average between the absolute upper bound and absolute lower bound is used. Given the absolute upper bound would be using the initial potato temperature and the absolutely lower bound would be using the room temperature, the average is found between these two.

$$h_{rad} = 2\sigma\varepsilon * (T_i^3 + T_{inf}^3) \quad \textbf{(6)}$$

Between the unwrapped and wrapped cases, the only difference is in the emittance. For the unwrapped case, the potato's emittance is approximately that of water ($\varepsilon_{water} = 0.96$) while in the wrapped case, the emittance is that of aluminum ($\varepsilon_{Al} = 0.09$). Using these values, the heat transfer coefficient for both wrapped and unwrapped cases be found. The unwrapped case yielded a value of $h_{rad,\ unwrapped} = 8.19\ W/m^2K$ while the value for the wrapped is $h_{rad,\ wrapped} = 0.76\ W/m^2K$. The total heat transfer coefficient is found by summing the natural convection and radiation coefficients. The table below summarizes the total heat transfer coefficients for the spherical/cylindrical and wrapped/unwrapped cases.

	Spherical	Cylindrical
Unwrapped	14.72 W/m^2K	14.31 W/m^2K
Wrapped	7.27 W/m^2K	6.86 W/m^2K

Lastly, the time constant must be found before the temperature can be predicted. Using equation (7), the time constant can be found for the cylindrical and spherical case. For the spherical case $\bar{L} = R/3$ and for the cylindrical case $\bar{L} = R/2$. The below table summarizes the time constant for all 4 combinations of wrapped/unwrapped and cylindrical/spherical

$$\tau = \rho_{potato} C_p \bar{L}/h \quad \textbf{(7)}$$

	Spherical	Cylindrical
Unwrapped	4.74E3	4.29E3
Wrapped	9.6E3	8.95E3

$$T(t) = T_{inf} + (T_i + T_{inf}) * exp(-t/\tau) \quad \textbf{(8)}$$

Finally, the dunking temperature expression can be used to predict the temperatures as time continues. The temperature expression is shown in equation (8). The temperature over time is shown below in figures 3.1 and 3.2.

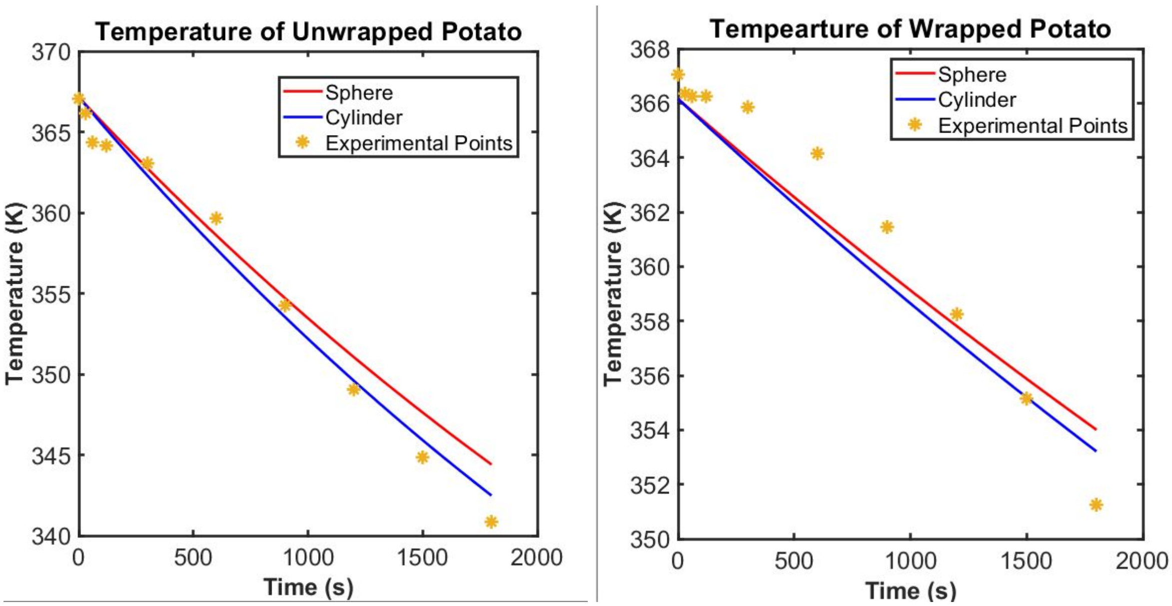

Figure 3.1 and 3.2

Both the cylindrical and spherical model offer approximately the same values for that follow the temperature of the potato closely. In the table below, the end temperatures in the cylindrical case, spherical case, and experimentally are shown for the wrapped and unwrapped potato.

Units (Kelvin)	Spherical	Cylindrical	Experimental
Wrapped	354	353.2	351.3
Unwrapped	344.4	342.5	340.8

Lastly, all assumptions previously made must be checked so as to ensure the model is appropriate. The assumptions that need to be checked are $Bi << 1$, the Rayleigh number must be $10^{-6} < Ra_D < 10^9$ for cylinder, and lastly that the surface temperature is quasistatic ($t^B < \tau$). The table below shows all these values for each case. As can be seen, all the assumptions are true. The biot numbers are not much less than one but are acceptable for an approximation. However this is a likely cause to some of the error found in the prediction. Other sources of error is the modeling of the potato as a linear system, meaning that the HTC for radiation and natural convection do not adjust as time goes on as they would in the experiment due to changing driving temperatures. An improvement to the mathematical model would be to solve for the HTC parallel with the temperature of the potato, such that it accurately updates given new parameters.

	Cylindrical	Spherical
Biot Unwrapped	0.33	0.38
Biot Wrapped	0.16	0.19
Rayleigh	1.5E6	7.5E6
Boundary Layer forming time	0.065	0.085

Parameters used:

Properties of Air	**Value**
ν	$1.851E-5\ [m^2/s]$
α	$2.616E-5\ [m^2/s]$
Pr	0.71
K	$0.02821\ [W/mK]$
Properties of Potato	
K	$0.66\ [W/mK]$
ρ	$967.4\ [kg/m^3]$
C	$4190\ [J/kgK]$
ε	0.96
Properties of Aluminum Foil	
K	$164\,[W/mK]$
ρ	$2781\,[kg/m^3]$
C	$883\ [J/kgK]$
ε	0.09
Parameters	
T_{inf}	295.15 K
T_i	367.15 K
g	$9.8\ [m/s^2]$
β	1/295.15 [1/K]
σ	$5.67E-8$